

TM5 PR. 9.54

A ROCKET STARTS FROM REST BY EMITTING MASS. AT WHAT FRACTION OF THE INITIAL MASS IS THE MOMENTUM A MAXIMUM?

FOR A ROCKET IN FREE SPACE (INITIALLY AT REST)

$$v = v_0^0 + u \ln\left(\frac{m_0}{m}\right)$$

WRITE THE MOMENTUM AND TAKE THE DERIVATIVE.

$$p = mu \ln\left(\frac{m_0}{m}\right)$$

$$\frac{dp}{dm} = u \ln\left(\frac{m_0}{m}\right) + mu \left(\frac{m}{m_0}\right) \left(\frac{-m_0}{m^2}\right)$$

SET EQUAL TO ZERO TO FIND THE MAXIMUM

$$\Rightarrow u \ln\left(\frac{m_0}{m_{\max}}\right) = \cancel{mu} \left(\frac{m_{\max}}{m_0}\right) \left(\frac{-m_0}{m_{\max}^2}\right)$$

$$\ln\left(\frac{m_0}{m_{\max}}\right) = 1$$

$$\Rightarrow \frac{m_0}{m_{\max}} = e$$

$$\text{OR } \boxed{m_{\max} = \frac{m_0}{e} = 0.37 m_0}$$

MAXIMUM MOMENTUM IS AT THE " $\frac{1}{e}$ " VALUE!